HOMOGENEOUS CAVITATION MODELING – ANALYSIS OF BASICS OF MATHEMATICAL FORMULATION OF SOURCE TERMS

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Summary

Cavitation is a phenomenon of fluid vaporization in the areas where the static fluid pressure drops below the saturated liquid pressure. The topic of this article is presentation of the physical basics of mathematical formulations of source terms of the chosen homogeneous models of cavitation phenomenon, i.e. Schnerr and Sauer (2001), Singhal et al. (2002), and Zwart et al. (2004), including the conversions leading to the final form of source terms, which are expressed in terms of mass and the analyzed constituent is vapor. The aim of the article is showing the similarities and differences at the selected stages of derivations of the mathematical formulas proposed by the chosen authors. The motivation to undertake the analysis of the mathematical basics of the source terms is literature lack of any works including a report of successive steps, along with the suitable comments allowing easy understanding of their final form.

Keywords: cavitation, source terms, homogeneous approach

MODELOWANIE HOMOGENICZNE ZJAWISKA KAWITACJI – ANALIZA PODSTAW MATEMATYCZNYCH SFORMUŁOWAŃ CZŁONÓW ŹRÓDŁOWYCH

Streszczenie

Kawitacja jest zjawiskiem parowania płynu w obszarach, w których ciśnienie statyczne płynu spada poniżej ciśnienia nasycenia. Tematem niniejszego artykułu jest przedstawienie fizycznych podstaw matematycznych sformułowań członów źródłowych wybranych homogenicznych modeli zjawiska kawitacji, Schnerra i Sauera (2001), Singhala i in. (2002) oraz Zwarta i in. (2004) z uwzględnieniem przekształceń prowadzących do uzyskania ostatecznej formy członów źródłowych, w której zapis wykonany jest w ujęciu masowym, zaś analizowanym składnikiem jest para. Celem artykułu jest przedstawienie podobieństw i różnic na kolejnych etapach wyprowadzeń wzorów matematycznych proponowanych przez różnych autorów. Motywacją do podjęcia się zagadnienia analizy podstaw matematycznych członów źródłowych jest brak w literaturze pozycji zawierających zestawienia następujących po sobie kroków wraz z odpowiednim komentarzem ułatwiającym zrozumienie ich ostatecznej formy.

Słowa kluczowe: kawitacja, człony źródłowe, podejście homogeniczne

NOMENCLATURE

C – empirical model constant

e – energy, $(\mathrm{kg} \cdot \mathrm{m}^2)/\mathrm{s}^2$

f – mass fraction

k – turbulence kinetic energy, $\mathrm{m}^2/\mathrm{s}^2$

$\dot{m}$ – mass transfer rates, mass source, $(\mathrm{kg}/(\mathrm{m}^3 \cdot \mathrm{s})$

$\dot{n}$ – volume transfer rates, mass source, $(1/(\mathrm{m}^3 \cdot \mathrm{s})$

N_B – number of bubbles
n_0 – number of cavitation nuclei per unit volume
p – local fluid pressure, Pa
p_{sat} – saturated vapour pressure, Pa
pI – spherical stress tensor, Pa
q^m – molecular heat flux, kg/s^3
q^R – turbulent heat flux, kg/s^3
R – bubble radius, m
s_b – intensity of the mass forces source, N/m^3
s_e – intensity of the energy source, J/(m^3/s)
t – time, s
u – velocity, m/s
u_{ref} – reference velocity, m/s
V – volume, m^3
We – the Weber number

Greek symbols

α - volume fraction
μ – dynamic viscosity, Pa · s
ρ – density, kg/m^3
σ – surface tension, N/m
τ^m – viscous molecular stress tensor, Pa
τ^R – turbulent Reynolds stress tensor, Pa

Subscripts and superscripts

d – evaporation rate
g - gas
l – liquid
m – mixture
nuc – nucleation site
p – condensation rate
v - vapour

1. INTRODUCTION

Cavitation is a physical non-equilibrium phenomenon of liquid evaporation in areas where the static local fluid pressure drops below the saturated vapor pressure. Cavitated areas are areas of turbulent flow of liquid stream. For the first time, this term was used in 1985, in relation to the air bubbles appearing in water around a screw propeller [16]. Since 1935, experimental investigations of cavitating flow have been conducted [4]. Initially, only a behavior of a single bubble was analyzed in numerical calculations. Over time, whole systems were considered. Methods of numerical analyses of cavitation phenomenon are divided into two main groups: the first bases on the interphase dynamics [1], the other considers fluid as a multiphase mixture with an average density. Analysis of multiphase mixture using the homogeneous approach [3,5,8,12,13,17] assumes solving conservation equations [15] of mass

$$\frac{\partial \rho}{\partial t} + div(\rho\vec{u}) = 0, \qquad (1)$$

momentum

$$\frac{\partial}{\partial t}(\rho\vec{u}) + div(\rho\vec{u}\otimes\vec{u}) = div\left(-p\overleftrightarrow{I} + \overleftrightarrow{\tau}^m + \overleftrightarrow{\tau}^R\right) + \rho\vec{s}_b \qquad (2)$$

and energy

$$\frac{\partial}{\partial t}(\rho e) + div(\rho e\vec{u} + \rho\vec{u}) = div[(\overleftrightarrow{\tau}^m + \overleftrightarrow{\tau}^R)\vec{u} + \vec{q}^m + \vec{q}^R] + \rho\vec{s}_e, \qquad (3)$$

and an additional transport equation of the chosen constituent. The equation could be described for the whole liquid volume in terms of volume

$$\frac{\partial \alpha_v}{\partial t} + div(\alpha_v u) = \dot{n}^+ + \dot{n}^-, \qquad (4)$$

as well as in terms of mass

$$\frac{\partial \rho_v \alpha_v}{\partial t} + div(\rho_v \alpha_v u) = \dot{m}^+ + \dot{m}^-. \qquad (5)$$

Because

$$\alpha_v + \alpha_l = 1, \qquad (6)$$

the vapour volume fraction increase is equal to the decrease of liquid volume fraction

$$\frac{\partial \alpha_v}{\partial t} = -\frac{\partial \alpha_l}{\partial t}. \qquad (7)$$

It is also true for the mass fraction.

The additional transport equation contains two characteristic source terms describing evaporation and condensation process, which are functions of the saturated vapor pressure in local conditions. The phase transformation could be presented in the form of liquid mass exchange and here the liquid mass increases during the condensation process, when the local fluid pressure increases above the saturated vapor pressure, and decreases during the evaporation process, when the local fluid pressure drops below the saturated vapor pressure:

$$\dot{m} = \begin{cases} \dot{m}^+ & if \quad p > p_{sat} \\ \dot{m}^- & if \quad p < p_{sat} \end{cases}, \qquad (8)$$

The phase transformation could be also expressed in form of vapor volume exchange, and here the vapor volume decreases in the condensation process and increases in the evaporation process:

$$\dot{n} = \begin{cases} \dot{n}^- & if \quad p > p_{sat} \\ \dot{n}^+ & if \quad p < p_{sat} \end{cases}. \qquad (9)$$

The most common source terms are based on the Rayleigh equation

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 = \frac{p_{sat} - p}{\rho_l}, \qquad (10)$$

which describes the dynamic of a single bubble [10], and more precisely on the mathematical formula derived from this equation, which considers the dynamic of changes of bubble radius.

$$\frac{dR}{dt} = \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}. \qquad (11)$$

After sixty years Plesset and Prosperetti [9] presented an expanded form of the Rayleigh equation known as the Rayleigh-Plesset equation

$$R\frac{d^2R}{dt^2}+\frac{3}{2}\left(\frac{dR}{dt}\right)^2=\frac{p_{sat}-p}{\rho_l}-\frac{2\sigma}{\rho_l R}-4\frac{\mu_l}{\rho_l R}\frac{dR}{dt}, \qquad (12)$$

which considers surface tension and liquid dynamic viscosity.

The paper presents an analysis of relationships between physical correlations and mathematical descriptions striving to the description of the changes occurring in the mixture, which the authors of the homogeneous cavitation models considered when formulating their own proposals of the source terms of the transport equation. The large amount of proposals available in the literature, which over time has been still rising, is a proof that the attempt to write the cavitation phenomenon in the form of source terms of the transport equation is not as simple as it might seem. The starting point should be obvious and unambiguous, but as it turns out, these points are many. Taking on the challenge of giving the final form of source terms of the transport equation is connected not only with the knowledge of previous solutions but first of all with understanding of the logic of individual authors of homogeneous cavitation models. Most authors presenting their own proposals of source terms of the transport equation concentrate on its final form and omit, or describe in a laconic way, the logic that accompanies the formation of the model. In the available literature, there is no points in discussing the basics of source terms of the transport equation, even of the most important homogeneous cavitation models, which would be a very practical guide for those who want to present their own solutions. The aim of this article is to complete the existing gap in knowledge and to present the links between physical correlations and mathematical notations. In the article, the solution path for the selected models is shown. The article concludes with a summary including similarities and differences resulting from the analysis.

A brief description of each model, together with the final form of the source terms of the transport equation, are presented in the first part of the article. The next part of the article contains the description of the mathematical basics of the source terms of the transport equation and steps that are necessary to derive the final form of the chosen models. A comparison of the analyzed homogeneous cavitation models showing similarities and differences between them, as well as indication of the applied simplifications and resulting consequences, are presented in the last part of the article.

2. ANALYSIS OF BASICS OF MATHEMATICAL FORMULAS OF THE SOURCE TERMS

The analysis of mathematical basics of source terms of the transport equation is conducted for the following models: Schnerr and Sauer, Singhal et al. and Zwart et al.

The Schnerr and Sauer model was presented in 2001 [12]. The distinguish feature of this model is a lack of the empirical constants characteristic for other models. It is based solely on the quantitative values of physical parameters. The source terms of the transport equation are formulated as follows:

$$\dot{m}^+=\frac{\rho_v\rho_l}{\rho_m}\alpha_v(1-\alpha_v)\frac{3}{R}\sqrt{\frac{2}{3}\frac{(p-p_{sat})}{\rho_l}} \qquad (13)$$

$$\dot{m}^-=-\frac{\rho_v\rho_l}{\rho_m}\alpha_v(1-\alpha_v)\frac{3}{R}\sqrt{\frac{2}{3}\frac{(p_{sat}-p)}{\rho_l}}. \qquad (14)$$

The Singhal et al. cavitation model [14], called the Full Cavitation model, owes its name to taking into consideration the large amount (in relation to other models) of physical parameters in the source terms. These physical parameters include local turbulence kinetic energy, mass fraction of non-condensable gases and surface tension. This model was presented in 2002 and was the first homogeneous cavitation model used commercially. The source terms of the transport equation in the Singhal et al. cavitation model are formulated as follows:

$$\dot{m}^+=C_p\frac{\sqrt{k}}{\sigma}\rho_l\rho_l\sqrt{\frac{2}{3}\frac{(p-p_{sat})}{\rho_l}} \qquad (15)$$

$$\dot{m}^-=-C_d\frac{\sqrt{k}}{\sigma}\rho_l\rho_v\sqrt{\frac{2}{3}\frac{(p_{sat}-p)}{\rho_l}}\left(1-f_v-f_g\right). \qquad (16)$$

The next examined model is the Zwart et al. model, which was presented in 2004 [17]. A distinguish feature of this model is replacement of the vapor volume fraction (α_v) with the product of the nucleation site of volume fraction (α_{nuc}) and the remaining fluid volume fraction (1- α_v) in the evaporation rate. The source terms in the transport equation of the Zwart et al. model are formulated as follows:

$$\dot{m}^+=C_p\frac{3\alpha_v\rho_v}{R}\sqrt{\frac{2}{3}\frac{(p-p_{sat})}{\rho_l}} \qquad (17)$$

$$\dot{m}^-=-C_d\frac{3(1-\alpha_v)\alpha_{nuc}}{R}\sqrt{\frac{2}{3}\frac{(p_{sat}-p)}{\rho_l}}. \qquad (18)$$

2.1. THE SCHNERR AND SAUER MODEL

The starting point of the source terms of the Schnerr and Sauer model [12] is the vapor volume fraction, understood as the vapor volume divided by the volume of vapor and liquid

$$\alpha_v=\frac{V_v}{V_m}, \qquad (19)$$

where the vapor volume is nothing other than the number of bubbles multiplied by the volume of the ball

$$V_v = N_B \cdot \frac{4}{3}\pi R^3, \tag{20}$$

and the volume of the mixture is the sum of the vapor volume and the liquid volume

$$V_m = V_l + V_v. \tag{21}$$

Thus, the vapor volume fraction can be written as follows:

$$\alpha_v = \frac{V_v}{V_m} = \frac{N_B\frac{4}{3}\pi R^3}{V_v+V_l}. \tag{22}$$

The number of bubbles in a given volume is dependent on the amount of cavitation nuclei in a given volume

$$N_B = n_0 V_l. \tag{23}$$

Consequently, the vapor volume fraction can be represented as

$$\alpha_v = \frac{V_v}{V_m} = \frac{n_0 V_l \frac{4}{3}\pi R^3}{n_0 V_l \frac{4}{3}\pi R^3 + V_l} = \frac{n_0 \frac{4}{3}\pi R^3}{n_0 \frac{4}{3}\pi R^3 + 1}, \tag{24}$$

and the change of vapor volume fraction in the time as

$$\frac{d\alpha_v}{dt} = \frac{n_0 4\pi R^2 \frac{dR}{dt} \left(n_0 \frac{4}{3}\pi R^3 + 1\right) - n_0 \frac{4}{3}\pi R^3 \cdot n_0 4\pi R^2 \frac{dR}{dt}}{\left(n_0 \frac{4}{3}\pi R^3 + 1\right)^2} = \frac{n_0 \cdot 4\pi R^2 \frac{dR}{dt}}{\left(n_0 \frac{4}{3}\pi R^3 + 1\right)^2}. \tag{25}$$

The liquid volume together with vapor volume is the volume of the mixture

$$V_m = V_l + V_v, \tag{26}$$

therefore, the liquid volume is nothing other than the difference between the mixture volume and the vapor volume

$$V_l = V_m - V_v. \tag{27}$$

Finally, the liquid volume could be defined as follows:

$$V_l = V_m - \frac{\alpha_v}{V_m} = V_m(1 - \alpha_v). \tag{28}$$

It results unambiguously from this formula that the difference between the whole fluid volume and the vapor volume fraction is

$$1 - \alpha_v = \frac{V_l}{V_m} = \frac{V_l}{V_v + V_l} = \frac{1}{1 + n_0 \frac{4}{3}\pi R^3}. \tag{29}$$

After inserting the relationship from the Eq. 29 into Eq. 25, the change of vapor volume fraction has a following form:

$$\frac{d\alpha_v}{dt} = (1 - \alpha_v) \frac{n_0 \cdot 4\pi R^2}{1 + n_0 \frac{4}{3}\pi R^3} \frac{dR}{dt}. \tag{30}$$

After transition from the volume fraction to the mass fraction the Eq. 30 is expressed as follows:

$$\frac{d\alpha_v}{dt} = \frac{\rho_m}{\rho_v \rho_l}(\dot{m}^+ + \dot{m}^-). \tag{31}$$

Additionally, considering the transition of the Eq. 24 by multiplying the equation by three and dividing by R

$$\frac{3\alpha_v}{R} = \frac{3}{R} \cdot \frac{n_0 \frac{4}{3}\pi R^3}{n_0 \frac{4}{3}\pi R^3 + 1} = \frac{n_0 \cdot 4\pi R^2}{n_0 \frac{4}{3}\pi R^3 + 1}, \tag{32}$$

the Eq. 31 takes the following form:

$$\dot{m}^+ + \dot{m}^- = \frac{\rho_v \rho_l}{\rho_m}(1 - \alpha_v) \frac{3\alpha_v}{R} \frac{dR}{dt}. \tag{33}$$

The final form of the equation, resulting from taking into account also the derivative of the R radius based on the Eq. 11, is as follows:

$$\dot{m}^+ + \dot{m}^- = \frac{\rho_v \rho_l}{\rho_m}(1 - \alpha_v) \frac{3\alpha_v}{R} \sqrt{\frac{2}{3} \frac{(p_{sat} - p)}{\rho_l}}. \tag{34}$$

Moreover, also the R radius appearing in the formula should be expressed as the function of the vapor volume through the transformation of the Eq. 24

$$\alpha_v \left(n_0 \cdot \frac{4}{3}\pi R^3 + 1\right) = n_0 \cdot \frac{4}{3}\pi R^3. \tag{35}$$

After multiplying of the vapor volume fraction by the expressions in brackets

$$n_0 \cdot \frac{4}{3}\pi R^3 - \alpha_v \cdot \left(n_0 \cdot \frac{4}{3}\pi R^3\right) = \alpha_v \tag{36}$$

and then factoring out the R radius

$$R^3 (1 - \alpha_v) \cdot \left(n_0 \cdot \frac{4}{3}\pi\right) = \alpha_v, \tag{37}$$

the radius could be write as follows:

$$R^3 = \frac{\alpha_v}{(1 - \alpha_v) \cdot \left(n_0 \frac{4}{3}\pi\right)} = \frac{3\alpha_v}{(1 - \alpha_v) \cdot (n_0 \cdot 4\pi)}. \tag{38}$$

The final expression of the radius R is

$$R = \left(\frac{3\alpha_v}{(1 - \alpha_v) \cdot (n_0 \cdot 4\pi)}\right)^{\frac{1}{3}}. \tag{39}$$

After insertion of the Eq. 39 to the Eq. 34, we obtain the final version of the equation describing the change of liquid mass fraction according to the Schnerr and Sauer model for condensation

$$\dot{m}^+ = \frac{\rho_v \rho_l}{\rho_m}(1 - \alpha_v) \frac{3\alpha_v}{\left(\frac{3\alpha_v}{(1 - \alpha_v) \cdot (n_0 \cdot 4\pi)}\right)^{\frac{1}{3}}} \cdot \sqrt{\frac{2}{3} \frac{(p - p_{sat})}{\rho_l}} \tag{40}$$

and evaporation

$$\dot{m}^- = -\frac{\rho_v \rho_l}{\rho_m}(1 - \alpha_v) \frac{3\alpha_v}{\left(\frac{3\alpha_v}{(1 - \alpha) \cdot (n_0 \cdot 4\pi)}\right)^{\frac{1}{3}}} \cdot \sqrt{\frac{2}{3} \frac{(p_{sat} - p)}{\rho_l}}. \tag{41}$$

2.2. THE SINGHAL ET AL. MODEL

For the authors of the Singhal et al. model [14], the starting point for the formulation of the source terms of the transport equation is also the change of the vapor volume fraction, as in the case of the Schnerr and Sauer model, but not due to the amount of the number of cavitation nuclei but to the mixture density

$$\rho_m = \alpha_v \rho_v + (1 - \alpha_v)\rho_l = \alpha_v(\rho_v - \rho_l) + \rho_l, \tag{42}$$

and more exactly from its derivative

$$\frac{\partial \rho_m}{\partial t} = -(\rho_l - \rho_v) \frac{\partial \alpha_v}{\partial t}. \tag{43}$$

The equation describing the vapor volume fraction

$$\alpha_v = \frac{V_v}{V_l} = \frac{n_0 V_l \frac{4}{3}\pi R^3}{V_l} = n_0 \cdot \frac{4}{3}\pi R^3 \quad (44)$$

another as used in the Schnerrr and Sauer model, is inserted to the mixture derivative. Unlike the Eq. 19, the vapor volume is divided by the liquid volume, not the mixture volume. The replacing the mixture volume with the liquid volume simplified significantly the formula for the vapor volume fraction but it does not reflect the real correlation. Into this derivative

$$\frac{\partial \rho_m}{\partial t} = -(\rho_l - \rho_v)\left(3 \cdot n_0 \cdot \frac{4}{3}\pi R^2 \frac{dR}{dt}\right) \quad (45)$$

the derivative of the bubble radius R from the Eq. 11 should be inserted

$$\frac{\partial \rho_m}{\partial t} = -(\rho_l - \rho_v) \cdot 4 n_0 \pi R^2 \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}. \quad (46)$$

The bubble radius R should be replaced by expression from the transformation of the Eq. 44 describing the vapor volume fraction, as the product of the bubbles number per unit volume and the volume of the sphere

$$R^3 = \frac{3\alpha_v}{4\pi n_0}, \quad (47)$$

so

$$R = \left(\frac{3\alpha_v}{4\pi n_0}\right)^{\frac{1}{3}} \quad (48)$$

and the Eq. 46 takes the following form

$$\frac{\partial \rho_m}{\partial t} = -(\rho_l - \rho_v) \cdot 4 n_0 \pi \left(\frac{3\alpha_v}{4\pi n_0}\right)^{\frac{2}{3}} \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}, \quad (49)$$

which after the arrangement is given as follows

$$\frac{\partial \rho_m}{\partial t} = -(\rho_l - \rho_v) \cdot (4 n_0 \pi)^{\frac{1}{3}} (3\alpha_v)^{\frac{2}{3}} \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}. \quad (50)$$

The Eq. 50 could be transformed after extracting of the bubble radius R into the following form, describing the mass change of the fluid during the evaporation

$$\dot{m}^- = -\frac{\rho_v \rho_l}{\rho_m} \cdot \frac{3\alpha_v}{R} \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}. \quad (51)$$

Considering, that the bubble radius R could be described using the following equation

$$R = \frac{0.061 \cdot We \cdot \sigma}{2\rho_l u_{ref}^2}, \quad (52)$$

the mass source term of the transport equation takes the following forms for evaporation

$$\dot{m}^- = -C_d \frac{\sqrt{k}}{\sigma} \rho_l \rho_v \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}} \left(1 - f_v - f_g\right), \quad (53)$$

and for condensation

$$\dot{m}^+ = C_p \frac{\sqrt{k}}{\sigma} \rho_l \rho_l \sqrt{\frac{2}{3}\frac{(p - p_{sat})}{\rho_l}}. \quad (54)$$

2.3. THE ZWART ET AL. MODEL

Considerations of the mathematical form of the source terms of the transport equation proposed by Zwart et al. [17] are based on very simple assumption that the change of the mixture mass results from the change of the mass of a single bubble multiplied by their number. The starting point here is the vapor volume fraction presented in the Eq. 44, i.e. the number of cavitation nuclei per unit volume multiplied by their volume. The change of the vapor volume fraction, that is, its derivative,

$$\frac{d\alpha_v}{dt} = n_0 \cdot 4\pi R^2 \frac{dR}{dt} \quad (55)$$

together with the derivative of the bubble radius from the Eq. 11 and the vapor density gives the mathematical form of the mass change given as

$$\dot{m} = n_0 \cdot 4\pi R^2 \rho_v \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}. \quad (56)$$

Additionally, the authors of the model decided to use the same definition of the vapor volume fraction as the authors of the Singhal et al. cavitation model and replaced the number of cavitation nuclei per unit volume (n_0) with the expression resulting from transformation of the Eq. 44

$$n_0 = \frac{3\alpha_v}{4\pi R^3}, \quad (57)$$

Finally, the Eq. 56 takes the following form:

$$\dot{m} = \frac{3\alpha_v}{4\pi R^3} \cdot 4\pi R^2 \rho_v \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}} = \frac{3\alpha_v}{R} \rho_v \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}. \quad (58)$$

The aim of this simplification is to remove the number of cavitation nuclei per unit volume, which is heavy to define, and to replace it with a physical quantity which is definitely easier to define in numerical calculations. The Eq. 58 shows the mathematical notation of the mass change resulting from the bubble growth, i.e. evaporation process. Considering the correction in the form of a proper ordering of the pressure values in the radius derivative from the Eq. 11 as well as introduction of an empirical constant, the mathematical description of the source terms for the condensation takes the following form:

$$\dot{m}^+ = C_p \frac{3\alpha_v \rho_v}{R} \sqrt{\frac{2}{3}\frac{(p - p_{sat})}{\rho_l}}. \quad (59)$$

Because of the instability of numerical calculations, the authors decided (in the case of the evaporation process) to replace the vapor volume fraction by the product of the nucleation site volume fraction (α_{nuc}) and the remaining fluid volume fraction ($1-\alpha_v$) and to introduce a new empirical constant

$$\dot{m}^- = -C_d \frac{3(1-\alpha_v)\alpha_{nuc}}{R} \sqrt{\frac{2}{3}\frac{(p_{sat} - p)}{\rho_l}}. \quad (60)$$

3. COMPARISON OF THE ANALYSED HOMOGENEOUS CAVITATION MODELS

From the conducted analysis of the Schnerr and Sauer, Singhal et al. and Zwart et al. models follows that in the case of the first model, i.e. the Schnerr and Sauer model, the starting point of the final form of the source terms is the vapor volume fraction defined as the ratio of the vapor volume to the volume of the mixture – Eq. 19, whereas the vapor volume is nothing other than the product of the bubbles number and their volume understood as the volume of the sphere – Eq. 20. The starting point in the Singhal et al. cavitation model is the analysis of the mixture density (Eq. 42), and more precisely its change, but in the Zwart et al. model it is the vapor volume fraction, and once again its change (Eq. 55), but using an another formula as in the Schnerr and Sauer model.

The analysis of the Schnerr and Sauer model, although it is the longest analysis covering the biggest number of equations, is the most transparent and leaves no room for doubt about the procedure. The transition from one stage to next stage is presented logically with the exception of the transition from the volume form (Eq. 30) to the mass form (Eq. 31). The standard formula considering the product of the vapor density and the fluid density divided by the mixture density is used here.

In the case of two other models, there are also ambiguities and inaccuracies. In the Zwart et al. model, in the equation defining the vapor volume fraction (Eq. 44) the vapor volume is divided by the liquid volume, not the mixture volume. The replacing the mixture volume with the liquid volume simplified significantly the formula for the vapor volume fraction but it does not reflect the real correlation. The consequence of this change can be necessity of replacement of the vapor volume fraction by the product of the nucleation site volume fraction (α_{nuc}) and the remaining fluid volume fraction ($1-\alpha_v$) as well as introduction of new empirical constants for condensation and evaporation to stabilize the numerical calculations.

The number of inaccuracies in the process of formulation of the source terms of the transport equation is the biggest in the Singhal et al. cavitation model. The change in the mixture density is used in the starting point, what in turn causes appearance of the difference between the liquid density and the vapor density (Eq. 43), which is then converted to the standard formula for the product of the liquid density and the vapor density divided by the mixture density applied in the Schnerr and Sauer model by the transition from the volume term (Eq. 30) to the mass term (Eq. 33). In the model was also used the same formula for the vapor volume fraction (Eq. 44) as in the Zwart et al. model. Additionally, using the Eq. 52 describing the bubble radius (R) is also incomprehensible. It is reduced only to the part containing the quotient of the surface tension to the square root of the turbulence kinetic energy. Any difficulties resulting from this simplification in numerical simulations are regulated by the introduction of the empirical constants for evaporation and condensation, as it was in the case of the Zwart et al. model.

Although all three analyzed models differ both in the starting point of the process of derivation of the mathematical dependencies of the source terms aimed at obtaining of the final form of the source terms of the transport equation as well as in the used simplifications, the common elements can be also indicated. These elements include the mathematical formula describing the dynamic of the bubble radius change (Eq. 11), as well as the assumption that the vapor volume is nothing other than the number of bubbles multiplied by the volume of the sphere (Eq. 20).

In two of the models presented, i.e. the Singhal et al. model and the Zwart et al. model, because of the applied simplifications, the empirical constants for evaporation and condensation were used. The values of the empirical constants vary depending on the analyzed system. It means that it is necessary to calibrate the model, if the data for the analyzed case is missing. Because of this difficulty, although the described models were tested many times [2,6,9], looking for a model, that would be universal for each system or even for a selected device class, is still ongoing.

Although the empirical constants for evaporation and condensation do not occur in the Schnerr and Sauer model, so it seems to be the ideal solution, it is not so obvious. The model was tested repeatedly [2,6] and it gained recognition, however results of numerical simulations obtained using this model do not ideally reflect results of experimental investigations. It should be born in mind that there are two more simplifications in the final form of the source terms of the transport equation (Eqs. 40 and 41) of the Schnerr and Sauer model than the assumption used to the transition from the volume term to the mass term. The first simplification concerns the equation describing the dynamic of the changes of the bubble radius (Eq. 11) resulting from the Rayleigh equation (Eq. 10) which is in itself a simplification. The use of the extended form of the more detailed Rayleigh-Plesset equation (Eq. 12) led to instability of numerical calculations, so the authors decided to leave the original solution. The next simplification, which is often omitted, is the volume of the bubble. There is no evidence that the bubble is an ideal ball. The third simplification concerns the distribution of the bubbles sizes. It is no taken into account, so the dynamic of the process can be strongly disturbed. Furthermore, it should be noted that in numerical simulations of fluid flow, many parameters should be considered, i.e. a turbulence model. To this day, the closure problem of turbulence models has not been solved unequivocally, and some proposals require using a very high-performance computer hardware, often unreachable for some researchers.

4. SUMMARY

The article presents the analysis of basics of the mathematical formulation of the source terms of the transport equation of three homogeneous cavitation models i.e. the Schnerr and Sauer model, Zwart et al. model and Singhal et al. model. The following observations could be made based on the performed analysis:

- each of the models has an another starting point;
- the starting point of the Schnerr and Sauer model is the vapor volume fraction;
- the starting point of the Singhal et al. model is the mixture density;
- the starting point of the Zwart et al. model is the vapor volume fraction but defined others than in the Schnerr and Sauer model;
- each of the analyzed models has two simplifications: the first is using of the mathematical formula describing the dynamic of the change of the bubble radius resulting from the Plesset equation, the second is the assumption of the bubble form as an ideal ball;
- the smallest number of simplifications and assumptions is in the Schnerr and Sauer model. Apart of two simplifications common for all the models, there is still the assumption used to the transition from the volume term to the mass term;
- the Singhal et al. model includes the biggest number of simplifications. Apart from the two simplifications common for all the models and as in the Schnerr and Sauer model, the assumption used to the transition from the volume term to the mass term, there is the simplification describing the bubble radius;
- the other simplifications are the distribution of the bubbles sizes, which is not taken into account, and a turbulence model.

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